

Indian Statistical Institute, Bangalore

B. Math (II)

First semester 2015-2016

Mid-Semester Examination : Statistics (I)

Date: 10-09-2015

Maximum Score 50

Duration: 3 Hours

1. The Pareto distribution with shape parameter $\alpha > 0$ and scale parameter $\beta > 0$ has probability density function (*pdf*)

$$f(x|\alpha, \beta) = \alpha\beta^\alpha x^{-(\alpha+1)} I_{(\beta, \infty)}(x)$$

Based on a random sample X_1, X_2, \dots, X_n , of size n , from $f(x|\alpha, \beta)$, obtain *method of moments (MOM) estimators* for α, β . Also obtain *maximum likelihood estimators (MLE)* for α, β .

[15]

2. Let $Z \sim N(0, 1)$ and $Y \sim \chi_n^2$ be independent. Define $T = \frac{Z}{\sqrt{\frac{Y}{n}}}$. Obtain the joint density function $f_{TY}(t, y)$ of T and Y . Hence obtain the marginal density function $f_T(t)$ of T and identify it.

[10]

3. The data below show IQ scores for 30 sixth graders.

088	102	126	095	109	099	102	151
115	097	092	107	081	119	094	090
109	099	102	117	098	093	105	084
114	122	087	094	101	081.		

- (a) Make a stem and leaf plot of these data.
(b) Find the sample mean \bar{X} .
(c) Find 100 p -th percentile for $p = 0.25, 0.50$ and 0.75 .
(d) Find the first quartile Q_1 , median M and the third quartile Q_3 .
(e) Draw the box plot and identify the outliers.
(f) Decide on trimming fraction just enough to eliminate the outliers and obtain the trimmed mean \bar{X}_T .
(g) Explain how to obtain the trimmed standard deviation S_T .
(h) Between the box plot and the stem and leaf plot what do they tell us about the above data set? In very general terms what can you say about the population from which the data arrived?

[3 + 1 + 3 + 1 + 4 + 2 + 2 + 4 = 20]

4. How would you select a *digit* from the set of *digits* $\{1, 2, \dots, 6\}$, with equal probability, using a given coin with probability of heads equal to $\theta \in (0, 1)$?

[10]